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Effective Lagrangian of the $\not{R}$MSSM for neutrino mass generation

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Abstract

We derive the lepton number violating dimension-five and dimension-seven operators, relevant for neutrino mass generation, in the Minimal Supersymmetry Standard Model without R-parity (the $\not{R}$MSSM) by using the effective Lagrangian method. We keep all the possible CP violating phases, and we establish the general relationship between the high-scale parameters and the low-energy observals associated with the Standard Model particles. We study in a specific model the dependence of neutrino masses on the CP phases.

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I. INTRODUCTION

Neutrino physics measurements show that neutrinos change their flavour in oscillationary manner [1], implying that neutrinos have a mass. In the Standard Model (SM) of particle interactions neutrinos are strictly massless due to lepton number conservation and non-existence of right-handed neutrinos. The same is true for the most popular extension of the SM, the Minimal Supersymmetric Standard Model (MSSM) [2], where the lepton number conservation follows from the assumed R-parity symmetry. The R-parity of a field is defined as $R = (-1)^{3B+L+2s} = (-1)^{3(B-L)+2s}$, where B, L and s are the baryon number, lepton number and spin, respectively [3]. The conservation of R-parity is imposed in order to prohibit the proton from fast decay, which may happen if both B and L are broken. If one abandons the R-symmetry by allowing lepton number to be violated, but leaving baryon number unbroken, the MSSM can accommodate $|\Delta L| = 2$ Majorana masses for neutrinos [4].

The superpotential of the MSSM with broken R-parity and conserved baryon number (acronymed as $\not{R}$MSSM) is in the most general case of the following form:

$$W = W_{R_P} + W_{\not{R}_P} \tag{1}$$

with

$$\begin{aligned} W_{R} &= \epsilon_0 H_u H_d + h^e_{IJ} H_d L_I E^c_J + h^d_{IJ} H_d Q_I D^c_J \\ &\quad + h^u_{IJ} H_u Q_I U^c_J \ , \\ W_{\not{R}} &= \epsilon_I H_u L_I + \frac{1}{2}\lambda_{IJK} L_I L_J E^c_K \\ &\quad + \lambda'_{IJK} L_I Q_J D^c_K \ . \end{aligned} \tag{2}$$

The part W_R is the superpotential of the standard R-conserving MSSM, whereas $W_{\not{R}}$ consists of renormalizable bilinear and trilinear couplings that violate the lepton number conservation and thereby the R-parity. In (2) H_u, H_d are Higgs doublet superfields and Q, L are left-handed quark and lepton superfields, all transforming as $SU(2)$ doublets, and U, D, E are right-handed quark and lepton superfields transforming as $SU(2)$ singlets. The flavour of the lepton and quark fields is denoted by indices I, J, K, and the summation over gauge indices is implicit. The R-parity violating three-lepton coupling obeys $\lambda_{IJK} = -\lambda_{JIK}$.

The breaking of supersymmetry is due to interaction and mass terms of component fields. In addition to the soft terms present in the MSSM, one should now include also the terms that break lepton number. The most general form of the soft-term Lagrangian in our case is then

$$\mathcal{L}_S = \mathcal{L}_S^R + \mathcal{L}_S^{\not R}, \tag{3}$$

where

$$\begin{aligned}
-\mathcal{L}_S^R = &(m_{\tilde{Q}}^2)_{IJ}\tilde{Q}_I^\dagger \tilde{Q}_J + (m_{\tilde{U}}^2)_{IJ}\tilde{U}_I^{c\dagger}\tilde{U}_J^c \\
&+(m_{\tilde{D}}^2)_{IJ}\tilde{D}_I^{c\dagger}\tilde{D}_J^c + (m_{\tilde{L}}^2)_{IJ}\tilde{L}_I^\dagger \tilde{L}_J \\
&+(m_{\tilde{E}}^2)_{IJ}\tilde{E}_I^{c\dagger}\tilde{E}_J^c + \Big[A_{IJ}^e H_d \tilde{L}_I \tilde{E}_J^c \\
&+A_{IJ}^d H_d \tilde{Q}_I \tilde{D}_J^c + A_{IJ}^u H_u \tilde{Q}_I \tilde{U}_J^c + h.c.\Big] \\
&+m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d \\
&+\Big(B_0 H_u H_d + h.c.\Big) + \Big[\frac{1}{2}m_1 \lambda_B \lambda_B \\
&+\frac{1}{2}m_2 \lambda_{A^\alpha}\lambda_{A^\alpha} + \frac{1}{2}m_3 \lambda_{G^a}\lambda_{G^a} + h.c.\Big]
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
-\mathcal{L}_S^{\not R} = &+\frac{1}{2}A_{IJK}\tilde{L}_I \tilde{L}_J \tilde{E}_K^c + A'_{IJK}\tilde{L}_I \tilde{Q}_J \tilde{D}_K^c \\
&+\mathbf{B}_I H_u \tilde{L}_I + (\mathbf{m}_{\tilde{L}}^2)_{0I} H_d^\dagger \tilde{L}_I + h.c.\ .
\end{aligned} \tag{5}$$

Here λ_{G^a} $(a = 1,\ 2,\ \cdots,\ 8)$, λ_{A^α} $(\alpha = 1,\ 2,\ 3)$, λ_B denote the gaugino fields corresponding to the gauge symmetry $SU(3) \times SU(2) \times U(1)$ in an obvious manner.

In this paper we shall study neutrino masses in the framework of the $\not R$MSSM. The induction of neutrino masses by R-parity violation couplings have been extensively analyzed in the literature [5]. Nevertheless, in most of these earlier studies the possible CP-phase effects on the neutrino masses have been ignored. In our analysis, we will keep all relevant CP violation phases of the Lagrangian as free parameters, and we shall show that they do affect the neutrino masses.

We will assume that the new physics, i.e. the physics that goes beyond the SM, is associated with an energy scale that is considerably larger than the electroweak scale of the SM, so that the new physics effects can be reliably analyzed by using the effective theory method. In that method, one integrates heavy fields out and keeps only the SM particles in

the resulting effective theory [6]. The effects of the heavy fields are parameterized in terms of high-dimensional operators of the light degrees of freedom. Due to the lepton number violating terms in the full Lagrangian, there will appear dimension-odd operators in the effective theory [7]. After the electroweak symmetry breaking, these operators will induce a small (Majorana) mass for neutrinos, in accordance with experimental results [8].

The paper is organized as follows. In Section II we will present preliminary considerations concerning Higgs field and fermion field doublets and define the SM Higgs doublet in terms of the original doublets and their vacuum expectation values. In Section III we derive, following the analysis of [9], the Wilson coefficients at the matching scale for the operators relevant for neutrino masses by integrating out the supersymmetric particles and the heavy Higgs doublet fields. The connection between the high-energy parameters and low-energy observables are also discussed in this Section. The results of numerical analysis and conclusions are presented in the last Section.

II. PRELIMINARIES

In the MSSM with no conserved lepton number there is no a priori distinction between the down-type Higgs boson and slepton doublets, which are assigned with identical quantum numbers. One can therefore freely rotate the weak eigenstate basis $(H_d, \tilde{L}_I)$ $(I = 1, 2, 3)$ by an $SU(4)$ transformation. The lepton number violating couplings depend on the basis one chooses. Nevertheless, the couplings of the SM Higgs field that appear in the low-energy theory should not depend on an specific choice of the basis.

The Higgs boson and slepton doublets can be presented as follows:

$$H_u = \begin{pmatrix} H_u^+ \\ \\ \frac{1}{\sqrt{2}}\left(\upsilon_u + H_u^0 + iA_u^0\right) \end{pmatrix},$$

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\upsilon_0 + H_d^0 + iA_d^0\right) \\ \\ H_d^- \end{pmatrix},$$

$$
\tilde{L}_{I} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Big(\upsilon_{I} + H^{0}_{I} + iA^{0}_{I}\Big) \\ \\ \tilde{L}^{-}_{I} \end{pmatrix} \quad (I = 1,\ 2,\ 3)\ . \tag{6}
$$

Here $\upsilon_{u},\ \upsilon_0,\ \upsilon_{I}$ denote the vacuum expected values (VEVs) of the neutral components of the doublets $H_u,\ H_d$ and $\tilde{L}_{I}$, respectively. One is obviously free to rotate these doublets in such a way that only one doublet achieves a non-vanishing vacuum expectation value. Let us denote

$$
\begin{aligned}
\upsilon_{d_0} &= \upsilon_{0}\ , \\
\upsilon_{d_1} &= \sqrt{\upsilon_{0}^2 + \upsilon_{1}^2}\ , \\
\upsilon_{d_2} &= \sqrt{\upsilon_{0}^2 + \upsilon_{1}^2 + \upsilon_{2}^2}\ , \\
\upsilon_{d_3} &= \sqrt{\upsilon_{0}^2 + \upsilon_{1}^2 + \upsilon_{2}^2 + \upsilon_{3}^2} \equiv \upsilon_{d}\ ,
\end{aligned} \tag{7}
$$

and replace the Higgs boson and slepton doublets of Eq. (6) with the set $(\Phi,\ \Phi_{H_1},\ \Phi_{H_2},\ \Phi_{H_3},\ \Phi_{H_4})$ defined as

$$
\begin{pmatrix} \Phi \\ \Phi_{H_1} \\ \Phi_{H_{I+1}} \end{pmatrix} = \mathcal{Z}^0_{H} \begin{pmatrix} H_{u} \\ (i\sigma^2)H^{*}_{d} \\ (i\sigma^2)\tilde{L}^{*}_{I} \end{pmatrix}\ , \tag{8}
$$

where $\mathcal{Z}^0_{H}$ denotes the transformation matrix

$$
\mathcal{Z}^0_{H} = \begin{pmatrix}
-s_{\beta} & c_{\beta}\frac{\upsilon_0}{\upsilon_d} & c_{\beta}\frac{\upsilon_1}{\upsilon_d} & c_{\beta}\frac{\upsilon_2}{\upsilon_d} & c_{\beta}\frac{\upsilon_3}{\upsilon_d} \\
c_{\beta} & s_{\beta}\frac{\upsilon_0}{\upsilon_d} & s_{\beta}\frac{\upsilon n_1}{\upsilon_d} & s_{\beta}\frac{\upsilon_2}{\upsilon_d} & s_{\beta}\frac{\upsilon_3}{\upsilon_d} \\
0 & -\frac{\upsilon_1}{\upsilon_{d_1}} & \frac{\upsilon_0}{\upsilon_{d_1}} & 0 & 0 \\
0 & -\frac{\upsilon_0\upsilon_2}{\upsilon_{d_1}\upsilon_{d_2}} & -\frac{\upsilon_1\upsilon_2}{\upsilon_{d_1}\upsilon_{d_2}} & \frac{\upsilon_{d_1}}{\upsilon_{d_2}} & 0 \\
0 & -\frac{\upsilon_0\upsilon_3}{\upsilon_{d_2}\upsilon_d} & -\frac{\upsilon_1\upsilon_3}{\upsilon_{d_2}\upsilon_d} & -\frac{\upsilon_2\upsilon_3}{\upsilon_{d_2}\upsilon_d} & \frac{\upsilon_{d_2}}{\upsilon_d}
\end{pmatrix}\ . \tag{9}
$$

The doublets $\Phi, \Phi_{H_\alpha},\ (\alpha = 1,\ 2,\ 3,\ 4)$ can be presented in the form

$$
\Phi = \begin{pmatrix} G^{+} \\ \\ \frac{1}{\sqrt{2}}\Big(h + \upsilon + iG^0\Big) \end{pmatrix}\ ,
$$

$$\Phi_{H_\alpha} = \begin{pmatrix} H_\alpha^+ \\ \\ \frac{1}{\sqrt{2}}\Big(H_\alpha^0 + iA_\alpha^0\Big) \end{pmatrix} . \tag{10}$$

The doublet Φ corresponds to the Higgs doublet of the SM, having a vacuum expectation value $\upsilon = \sqrt{\upsilon_u^2 + \upsilon_d^2} = 246$ GeV. It remains massless prior to the electroweak symmetry breaking and does not mix with the other doublets. The other doublets obtain a large mass originating in the soft supersymmetry breaking terms of the Lagrangian. Indeed, the masses of the doublets are given by

$$\Big(\Phi \;\; \Phi^\dagger_{H_1} \;\; \Phi^\dagger_{H_{I+1}} \Big) \mathbf{M}^2_H \begin{pmatrix} \Phi \\ \Phi_{H_1} \\ \Phi_{H_{J+1}} \end{pmatrix} , \tag{11}$$

where

$$\mathbf{M}^2_H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (B/s_\beta c_\beta) & (s_\beta - c_\beta)E_0 E^*_I \\ 0 & (s_\beta - c_\beta)E^*_0 E_J & (\mathbf{m}^2_H)_{IJ} + E^*_I E_J \end{pmatrix} . \tag{12}$$

We have used here the following notations:

$$\begin{aligned}
B &= \sum_{\rho=0}^{3} \frac{\upsilon_\rho}{\upsilon_d}\Big(\mathbf{B}\Big)_\rho , \\
\Big(\mathbf{m}^2_H\Big)_{IJ} &= \frac{\upsilon_{d_{(I-1)}}\upsilon_{d_{(J-1)}}}{\upsilon_{d_I}\upsilon_{d_J}}\Big(\mathbf{m}^2_{\tilde{L}}\Big)_{IJ} \\
&\quad + \frac{\upsilon_I \upsilon_J}{\upsilon_{d_{(I-1)}}\upsilon_{d_{(J-1)}}\upsilon_{d_I}\upsilon_{d_J}} \sum_{\rho=0}^{I-1}\sum_{\sigma=0}^{J-1} \upsilon_\rho \upsilon_\sigma \Big(\mathbf{m}^2_{\tilde{L}}\Big)_{\rho\sigma} \\
&\quad - \frac{\upsilon_{d_{(J-1)}}\upsilon_I}{\upsilon_{d_{(I-1)}}\upsilon_{d_I}\upsilon_{d_J}} \sum_{\rho=0}^{I-1} \Big(\mathbf{m}^2_{\tilde{L}}\Big)_{J\rho}\upsilon_\rho \\
&\quad - \frac{\upsilon_{d_{(I-1)}}\upsilon_J}{\upsilon_{d_{(J-1)}}\upsilon_{d_I}\upsilon_{d_J}} \sum_{\rho=0}^{J-1} \upsilon_\rho \Big(\mathbf{m}^2_{\tilde{L}}\Big)_{\rho I} , \\
\mathbf{E}_0 &= \sum_{\rho=0}^{3} \frac{\upsilon_\rho}{\upsilon_d}\epsilon_\rho , \\
\mathbf{E}_I &= \frac{\upsilon^2_{d_{(I-1)}}\epsilon_I - \upsilon_I \sum_{\rho=0}^{I-1}\upsilon_\rho \epsilon_\rho}{\upsilon_{d_{(I-1)}}\upsilon_{d_I}} .
\end{aligned} \tag{13}$$

Here $I,\; J = 1,\; 2,\; 3$ and $\rho, \sigma = 0,\; 1,\; 2,\; 3$, and we have denoted $(\mathbf{m}^2_{\tilde{L}})_{00} = m^2_{H_d}$ and $(\mathbf{m}^2_{\tilde{L}})^*_{I0} = (\mathbf{m}^2_{\tilde{L}})_{0I}$. In the effective theory the heavy eigenstates of this mass Lagrangian are

integrated out and only the fields in the massless doublet Φ are considered as a physical degrees of freedom.

The bilinear R-parity violating terms of the Lagrangian (5) lead to mixing of the down-type higgsino and the left-handed leptons, and some of these fields obtain a large mass. The mass eigenstates are obtained by the following transformation:

$$\begin{pmatrix} \psi_{h_d} \\ \psi_{l_I} \end{pmatrix} = \mathcal{Z}_{\tilde{h}} \begin{pmatrix} \psi_{H_d} \\ \psi_{L_I} \end{pmatrix} \ (I = 1,\ 2,\ 3)\ , \tag{14}$$

where the transformation matrix $\mathcal{Z}_{\tilde{h}}$ is given by

$$\mathcal{Z}_{\tilde{h}} = \begin{pmatrix} \frac{|\epsilon_0|}{\mu_H} & \frac{|\epsilon_1|}{\mu_H}e^{i\varphi_1} & \frac{|\epsilon_2|}{\mu_H}e^{i\varphi_2} & \frac{|\epsilon_3|}{\mu_H}e^{i\varphi_3} \\ -\frac{|\epsilon_1|}{\mu_{H_1}}e^{-i\varphi_1} & \frac{|\epsilon_0|}{\mu_{H_1}} & 0 & 0 \\ -\frac{|\epsilon_0\epsilon_2|}{\mu_{H_1}\mu_{H_2}}e^{-i\varphi_2} & -\frac{|\epsilon_1\epsilon_2|}{\mu_{H_1}\mu_{H_2}}e^{-i(\varphi_2-\varphi_1)} & \frac{\mu_{H_1}}{\mu_{H_2}} & 0 \\ -\frac{|\epsilon_0\epsilon_3|}{\mu_{H_2}\mu_H}e^{-i\varphi_3} & -\frac{|\epsilon_1\epsilon_3|}{\mu_{H_2}\mu_H}e^{-i(\varphi_3-\varphi_1)} & -\frac{|\epsilon_2\epsilon_3|}{\mu_{H_2}\mu_H}e^{-i(\varphi_3-\varphi_2)} & \frac{\mu_{H_2}}{\mu_H} \end{pmatrix} . \tag{15}$$

Here the following notation is used:

$$\begin{aligned} \mu_H &= \sqrt{\sum_{\rho=0}^{3}|\epsilon_\rho|^2},\ \mu_{H_0} = |\epsilon_0|, \\ \mu_{H_1} &= \sqrt{|\epsilon_0|^2 + |\epsilon_1|^2}, \\ \mu_{H_2} &= \sqrt{|\epsilon_0|^2 + |\epsilon_1|^2 + |\epsilon_3|^2}, \\ \mu_{H_3} &= \mu_H. \end{aligned} \tag{16}$$

The phases φ_I $(I = 1,\ 2,\ 3)$, appearing in the transformation matrix, are defined as $\varphi_I = arg(\epsilon_I) - \theta_\mu$, where $\theta_\mu \equiv arg(\epsilon_0)$. The mass terms induced by the bilinear terms are

$$\mu_H e^{i\theta_\mu}\psi_{H_u}\psi_{h_d} + \mu_H e^{-i\theta_\mu}\overline{\psi}_{H_u}\overline{\psi}_{h_d} \tag{17}$$

The phase θ_μ can be absorbed into the fields by the following redefinitions:

$$\psi_{H_u} \to e^{-\frac{i}{2}\theta_\mu}\psi_{H_u},\ \psi_{h_d} \to e^{-\frac{i}{2}\theta_\mu}\psi_{h_d}. \tag{18}$$

One can then deduce that Eq. (17) is a mass term of the Dirac field $\tilde{h}^T = \left(\psi_{H_u}, \overline{\psi}_{h_d}\right)$. This heavy higgsino field, with a mass μ_H, is integrated out in the effective theory.

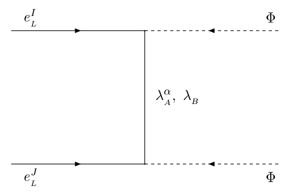

FIG. 1: The tree level diagrams that lead to the dimension-five operators.

III. THE EFFECTIVE LAGRANGIAN WITH DIMENSION-ODD OPERATORS

In the effective theory that results when the heavy fields are integrated out neutrino masses are in leading order generated by the following dimension-five operators consisting of the SM fields [7]:

$$\delta\mathcal{L}_{d=5} = \frac{1}{2}(C_1)_{J,I}\Big(\overline{(e^J_L)^c}\ \tilde{\Phi}^*\Big)\Big(\tilde{\Phi}^\dagger\ e^I_L\Big)$$
$$+\frac{1}{2}(C_2)_{J,I}\Big(\overline{(e^J_L)^c}\sigma^a\tilde{\Phi}^*\Big)\Big(\tilde{\Phi}^\dagger\sigma^a e^I_L\Big) + h.c., \tag{19}$$

where $(C_1)_{J,I}, (C_2)_{J,I}$ are Wilson coefficients. The tree-level Feynman diagrams inducing the above effective Lagrangian are given in Fig. 1. The Wilson coefficients corresponding to these diagrams are

$$(C_1)_{J,I} = \frac{g_1^2 c_\beta^2}{|m_1|}\Lambda_{0,I}\Lambda_{0,J}e^{-i\theta_1}\ ,$$
$$(C_2)_{J,I} = \frac{g_2^2 c_\beta^2}{|m_2|}\Lambda_{0,I}\Lambda_{0,J}e^{-i\theta_2}\ , \tag{20}$$

where $\tan\beta = \upsilon_u/\upsilon_d,\ c_\beta = \cos\beta,\ s_\beta = \sin\beta$, and the parameters $\Lambda_{I,J}$ are definited as

$$\Lambda_{0,0} = \frac{\upsilon_0|\epsilon_0|}{\upsilon_d\mu_H} + \sum_{I=1}^{3}\frac{\upsilon_I|\epsilon_I|}{\upsilon_d\mu_H}e^{-i\varphi_I}\ ,$$
$$\Lambda_{I,0} = \frac{\upsilon_{d_{(I-1)}}|\epsilon_I|}{\upsilon_{d_I}\mu_H}e^{-i\varphi_I} - \sum_{J=0}^{I-1}\frac{\upsilon_I\upsilon_J|\epsilon_J|}{\upsilon_{d_{(I-1)}}\upsilon_{d_I}\mu_H}e^{-i\varphi_J}\ ,$$
$$\Lambda_{0,I} = \frac{\upsilon_I\mu_{H_{(I-1)}}}{\upsilon_d\mu_{H_I}} - \sum_{J=0}^{I-1}\frac{|\epsilon_I\epsilon_J|e^{i(\varphi_I-\varphi_\beta)}\upsilon_J}{\upsilon_d\mu_{H_{(I-1)}}\mu_{H_I}} \tag{21}$$

with $I, J = 1,\ 2,\ 3$. When the EW symmetry breaks down, one ends up with the following mass matrix for neutrinos:

$$\mathbf{m}_\nu = \frac{1}{2}c_\beta^2\upsilon^2\Big(\frac{g_1^2 e^{-i\theta_1}}{|m_1|} + \frac{g_2^2 e^{-i\theta_2}}{|m_2|}\Big)$$

$$\times \begin{pmatrix} \Lambda_{0,1}^2 & \Lambda_{0,1}\Lambda_{0,2} & \Lambda_{0,1}\Lambda_{0,3} \\ \\ \Lambda_{0,1}\Lambda_{0,2} & \Lambda_{0,2}^2 & \Lambda_{0,2}\Lambda_{0,3} \\ \\ \Lambda_{0,1}\Lambda_{0,3} & \Lambda_{0,2}\Lambda_{0,3} & \Lambda_{0,3}^2 \end{pmatrix} . \tag{22}$$

This matrix is diagonalized by the unitary matrix

$$V = \begin{pmatrix} 0 & -\frac{A_1}{A} & \frac{\Lambda_{0,1}^*}{A} \\ \\ -\frac{\Lambda_{0,3}}{A} & \frac{\Lambda_{0,1}\Lambda_{0,2}^*}{AA_1} & \frac{\Lambda_{0,2}^*}{A} \\ \\ \frac{\Lambda_{0,2}}{A} & \frac{\Lambda_{0,1}\Lambda_{0,3}^*}{AA_1} & \frac{\Lambda_{0,3}^*}{A} \end{pmatrix} , \tag{23}$$

where

$$\begin{aligned} A &= \sqrt{|\Lambda_{0,1}|^2 + |\Lambda_{0,2}|^2 + |\Lambda_{0,3}|^2} , \\ A_1 &= \sqrt{|\Lambda_{0,1}|^2 + |\Lambda_{0,2}|^2} , \end{aligned} \tag{24}$$

giving

$$\begin{aligned} V^\dagger M_\nu V = &\frac{1}{4} c_\beta^2 \upsilon^2 \Big(\frac{g_1^2 e^{-i\theta_1}}{|m_1|} + \frac{g_2^2 e^{-i\theta_2}}{|m_2|} \Big) \\ &\times \mathrm{diag}(0,\ 0,\ A^2) . \end{aligned} \tag{25}$$

Hence only one neutrino flavor acquires a non-vanishing mass at tree level. This mass is given more explicitly by

$$\begin{aligned} m_{\nu_3} = &\frac{1}{2\mu_H^2} \Big| \frac{g_1^2 e^{-i\theta_1}}{|m_1|} + \frac{g_2^2 e^{-i\theta_2}}{|m_2|} \Big| \\ &\times \Big[\upsilon_d^2 \mu_H^2 - \Big| \upsilon_0 |\epsilon_0| + \sum_{I=1}^{3} \upsilon_I |\epsilon_I| e^{i\varphi_I} \Big|^2 \Big] . \end{aligned} \tag{26}$$

By setting the phases zero and taking account of the approximations used, we find our result coincident with that of Ref. [10].

An obvious conclusion from above is that the neutrino oscillation data, which indicates the existence of two distinctly different mass-difference scales, is not explained in the MSSM at tree level. Therefore, a study of higher order contributions to the neutrino mass Lagrangian is necessary.

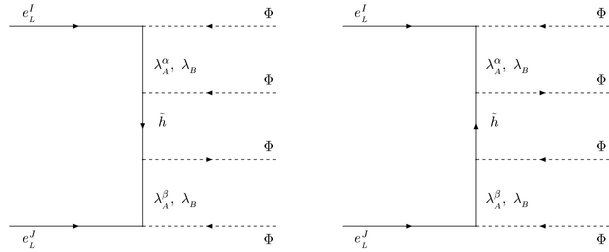

FIG. 2: The tree level diagrams that lead to the dimension-seven operators.

Let us first consider the corrections of dimension-seven operators to the neutrino mass matrix. The tree-level Wilson coefficients of the dimension-seven operatos and the Wilson coefficients of the dimension-five operators (19) at one-loop level belong to the same order of perturbative expansion. The dimension-seven operators that give corrections to the neutrino mass matrix are the following:

$$\begin{aligned}\delta\mathcal{L}_{d=7} = &\frac{1}{2}(C_3)_{J,I}\Big(\Phi^\dagger\Phi\Big)\Big(\overline{(e_L^J)^c}\;\tilde\Phi^*\Big)\Big(\tilde\Phi^\dagger\; e_L^I\Big)\\ &+\frac{1}{2}(C_4)_{J,I}\Big(\Phi^\dagger\sigma^a\Phi\Big)\Big(\overline{(e_L^J)^c}\tilde\Phi^*\Big)\Big(\tilde\Phi^\dagger\sigma^a e_L^I\Big)\\ &+\frac{1}{2}(C_5)_{J,I}\Big(\Phi^\dagger\sigma^a\Phi\Big)\Big(\overline{(e_L^J)^c}\sigma^a\tilde\Phi^*\Big)\Big(\tilde\Phi^\dagger e_L^I\Big)\\ &+\frac{1}{2}(C_6)_{J,I}\Big(\Phi^\dagger\sigma^a\sigma^b\Phi\Big)\Big(\overline{(e_L^J)^c}\sigma^a\tilde\Phi^*\Big)\Big(\tilde\Phi^\dagger\sigma^b e_L^I\Big)\\ &+h.c.\;,\end{aligned} \tag{27}$$

where the Wilson coefficients have, refering to Fig. 2, the expressions

$$\begin{aligned}(C_3)_{J,I} &= \frac{g_1^4}{|m_1|^2\mu_H}s_\beta c_\beta^3 e^{-i(\theta_\mu+2\theta_1)}\\ &\quad\times\Lambda_{0,0}\Lambda_{0,I}\Lambda_{0,J}\;,\\ (C_4)_{J,I} &= \frac{g_1^2g_2^2}{|m_1m_2|\mu_H}s_\beta c_\beta^3 e^{-i(\theta_\mu+\theta_1+\theta_2)}\\ &\quad\times\Lambda_{0,0}\Lambda_{0,I}\Lambda_{0,J}\;,\\ (C_5)_{J,I} &= (C_4)_{J,I}\;,\\ (C_6)_{J,I} &= \frac{g_2^4}{|m_2|^2\mu_H}s_\beta c_\beta^3 e^{-i(\theta_\mu+2\theta_2)}\\ &\quad\times\Lambda_{0,0}\Lambda_{0,I}\Lambda_{0,J}\;.\end{aligned} \tag{28}$$

After the electroweak symmetry breaking these operators yield corrections to the neutrino mass matrix, which are accounted for by the following replacement in Eq. (22):

$$\Big(\frac{g_1^2e^{-i\theta_1}}{|m_1|}+\frac{g_2^2e^{-i\theta_2}}{|m_2|}\Big)$$

$$\longrightarrow \Big(\frac{g_1^2 e^{-i\theta_1}}{|m_{_1}|} + \frac{g_2^2 e^{-i\theta_2}}{|m_{_2}|}\Big)\Big[1 - \frac{1}{2} s_{_\beta} c_{_\beta} \Lambda_{_{0,0}}$$
$$\times \frac{\upsilon^2}{\mu_{_H}} e^{-i\theta_\mu} \Big(\frac{g_1^2 e^{-i\theta_1}}{|m_{_1}|} + \frac{g_2^2 e^{-i\theta_2}}{|m_{_2}|}\Big)\Big] \,. \tag{29}$$

The same replacement should be done in Eq. (26). We realize that when the tree-level contributions of the dimension-seven operators taken account, still only one neutrino flavour acquires a nonzero mass.

In Fig. 3 and Fig. 4 we present the one-loop diagrams that contribute to the Wilson coefficients of dimension-five operator after the matching procedure. In the full theory, the self-energy corrections to external leg, appearing in the diagrams in Fig. 3, should in principle include also the light-light contributions, such as $e_{_L} e_{_L}$, $\Phi\Phi$. However, such diagrams would not give any contribution to the dimension-five operators after the matching of the full and effective theory [9]. A similar conclusion is true also for the vertex correction diagrams that involve only the light degrees of freedom.

The lengthy expressions of the ensuing Wilson coefficients are presented in appendix A.

After the EW symmetry breaking, the neutrino mass matrix with the one-loop corrections included can be written as

$$M_{_\nu} + \delta M_{_\nu}, \tag{30}$$

where $M_{_\nu}$ denotes the contribution from the tree level dimension-five and dimension-seven operators and $\delta M_{_\nu}$ is the correction from the dimension-five operators at the one-loop level, given by

$$\Big(\delta M_{_\nu}\Big)_{_{J,I}} = \frac{\upsilon^2}{2}\Big(\delta C_{_1}^{(1)} + \delta C_{_2}^{(1)} + \delta C_{_1}^{(2)}$$
$$+ \delta C_{_2}^{(2)}\Big)_{_{J,I}} \,, \tag{31}$$

where the quantities $\delta C_{_{1,2}}^{(1,2)}$ are defined in appendix A.

The neutrino mass matrix is diagonalized, as usual, by a unitary transformation

$$U^\dagger \Big(M_{_\nu} + \delta M_{_\nu}\Big) U = \mathrm{diag}(m_{_{\nu_1}},\ m_{_{\nu_2}},\ m_{_{\nu_3}}) \,, \tag{32}$$

where the 3×3 unitary matrix U generally contains 3 rotation angles and three physical phases (CP-phases), and $m_{_{\nu_i}}$ denote the real positive mass eigenvalues of the light Majorana neutrinos. Among the three CP phases, two are so-called Majorana phases associated with

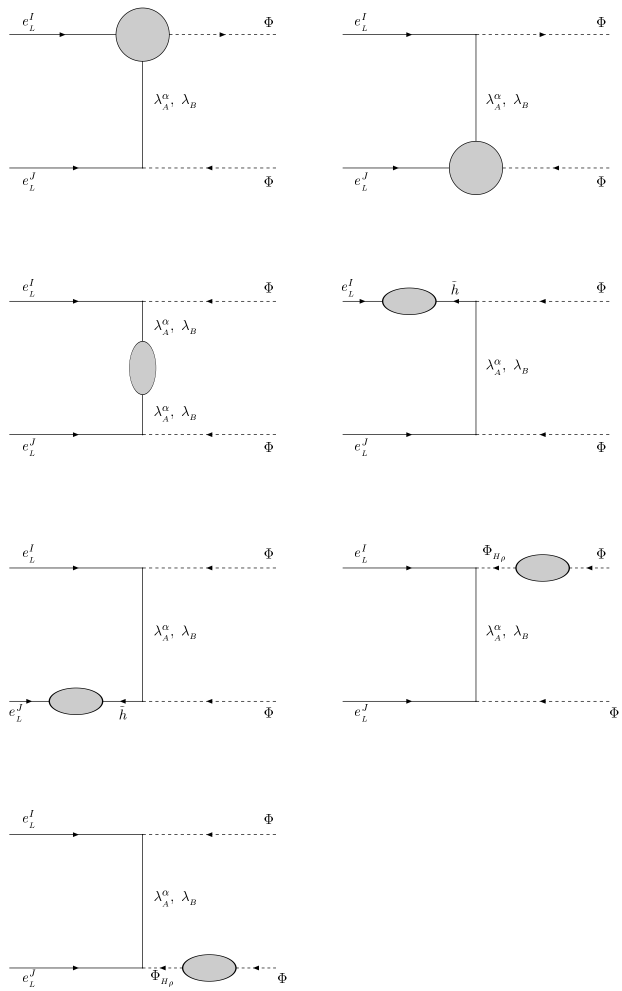

FIG. 3: The one-loop diagrams that lead to the dimension-five operators after the matching. The triangle, and self-energy diagrams denoted by blobs are presented in Figs. 5 to 8.

lepton number violating terms in the mass Lagrangian and the third one is a so-called Dirac phase. One can factorize the Majorana phases by defining the transformation matrix U as

$$U = U_L D_m \ , \tag{33}$$

where U_L is the lepton mixing matrix, a counter part of the Cabibbo-Kobayashi-Maskawa mixing matrix of quarks, containing the Dirac phase, and D_m is a diagonal matrix with two Majorana phases. One can use the following parametrization:

$$D_m = \begin{pmatrix} e^{i\omega_1} & 0 & 0 \\ 0 & e^{i\omega_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \ , \tag{34}$$

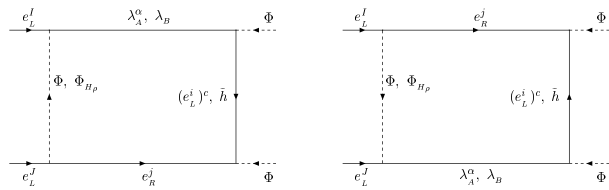

FIG. 4: The box-diagrams that lead to the dimension-five operators after the matching.

$$U_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{35}$$

with $c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$ and θ_{ij} denote the mixing angles.

The rotation angles θ_{ij} and the phases δ and ω_i appearing in the lepton mixing matrix U_L, as well as the values of neutrino masses, can be expressed in terms of the parameters of the the model. In the following we will study the constraints set by the atmospheric and solar neutrino oscillation data on the model parameters.

IV. CONSTRAINTS FROM OSCILLATION DATA

The neutrino oscillation data have outlined the mass and mixing pattern of neutrinos. According to a global fit to the data, the oscillations are best explained in terms of the following set of parameters [11]

$$\begin{aligned} &\Delta m^2_{32} = 2.0 \times 10^{-3}\ \mathrm{eV}^2, \\ &\Delta m^2_{21} = 7.2 \times 10^{-5}\ \mathrm{eV}^2, \\ &\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{12} = 0.3, \\ &\sin^2\theta_{13} < 0.06\ . \end{aligned} \tag{36}$$

Let us now discuss the implications of these empirical results for the parameters of the $\not{R}$MSSM. Beside the parameters appearing in the MSSM, the $\not{R}$MSSM includes generally many new ones (in general complex) related to lepton number violation: three bilinear (ϵ_I) and thirty six trilinear (nine λ_{IJK}, twenty seven λ'_{IJK}) couplings in the superpotential, together with the corresponding soft breaking (three B_I) and A-terms (nine A_{IJK}, twenty seven

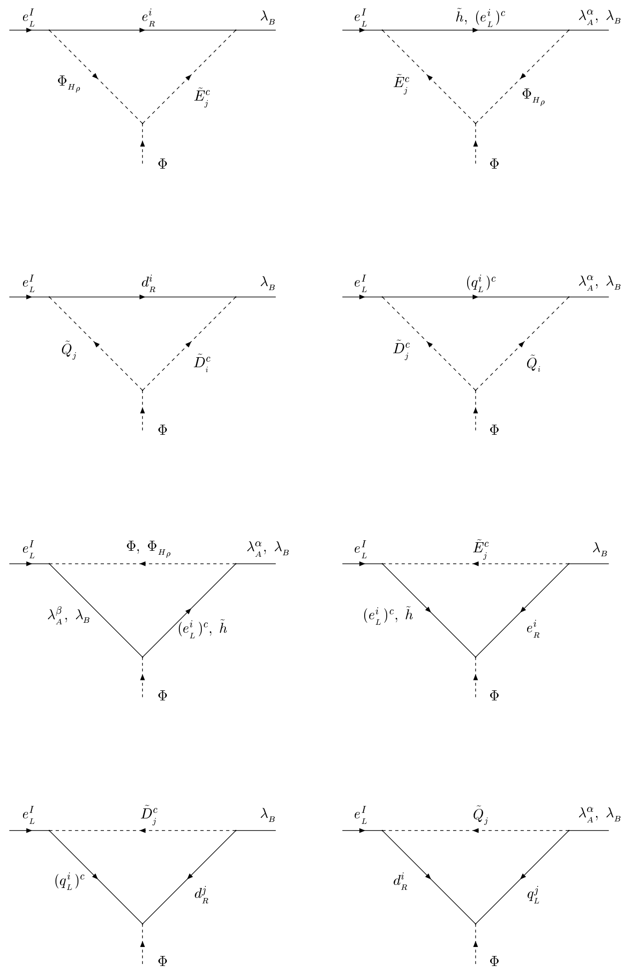

FIG. 5: The one-loop diagrams that lead to the dimension-five operators after the matching. The triangle-, and self energy-diagrams are presented in Figs. 5 to 7.

A'_{IJK}) and three additional $\not{R}$ soft masses $(m^2_{\tilde{L}})_{0I}$. There are only six physical parameters among the nine bilinear $\not{R}$ parameters ($\epsilon_I,\ B_I,\ (m^2_{\tilde{L}})_{0I}$) due to the ambiguity in the choice of the $(H_d,\ \tilde{L})$ basis. It is obvious, giving the large number of parameters, the oscillation data alone would leave in the general case much free space for the choice of parameters. In the following we will not consider the most general situation but restrict ourselves to the case where only the bilinear $\not{R}$ couplings are present. We are particularly interested in the effects of the complex phases associated with these couplings.

Neutrino masses in the framework of $\not{R}$MSSM are extensively discussed in the literature, but the studies are usually restricted to the CP-conserving case leaving the possible effects of CP phases with less attention. In the MSSM, the soft broken terms provide new sources

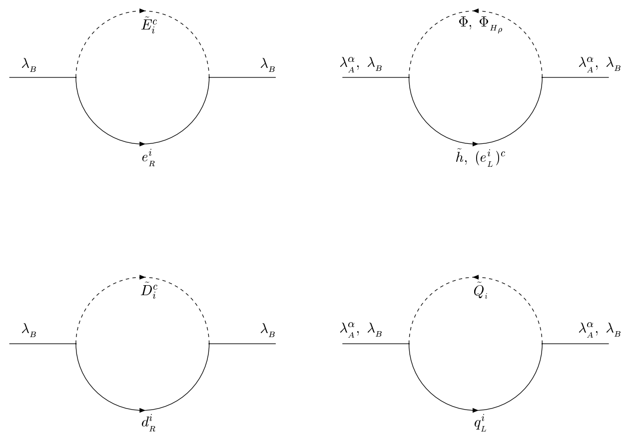

FIG. 6: The gaugino self-energy diagrams that lead to the dimension-five operators after the matching.

for the CP violation, in addition to the CKM mechanism of the Standard Model. At present, the strictest constraints on the CP phases originate from the experimental bounds on the electric dipole moments (EDM) of the electron and the neutron. Nevertheless, if one invokes a cancellation mechanism among different supersymmetric contributions [12] or choose the sfermions of two generations heavy enough [13], the loop-induced EDM's yield a bound only for the argument of the parameter μ, implying $\mu \leq \pi/(5\tan\beta)$, leaving the other explicit CP-violation phases unconstrained.

In principle, all the parameters associated with R-parity violation are complex in the model we are considering. For simplicity, we will now assume that all parameters are real except the gaugino masses. Furthermore, in order to apply the effective theory method safely, we assume the new physics scale is $\mu_{NP} = 10$ TeV, in other words, much higher than the electroweak scale. We choose the basis $\upsilon_I = 0$ $(I = 1,\ 2,\ 3)$, and we set

$$
\begin{aligned}
&(m_{\tilde{L}})_{IJ} = \mathrm{diag}(1,\ \sqrt{2},\ \sqrt{3}) \times 10\ \mathrm{TeV}, \\
&(m_{\tilde{L}})_{0I} = (100,\ 100,\ 100)\ \mathrm{GeV}, \\
&(m_{\tilde{E}})_{IJ} = (m_{\tilde{U}})_{IJ} = (m_{\tilde{D}})_{IJ} = \mathrm{diag}(5,\ 5,\ 1)\ \mathrm{TeV}, \\
&(m_{\tilde{Q}})_{IJ} = \mathrm{diag}(5,\ 5,\ 1)\ \mathrm{TeV}, \\
&A^u_{IJ} = A^d_{IJ} = A^e_{IJ} = 0\ , (I,\ J = 1,\ 2,\ 3)\ .
\end{aligned}
$$

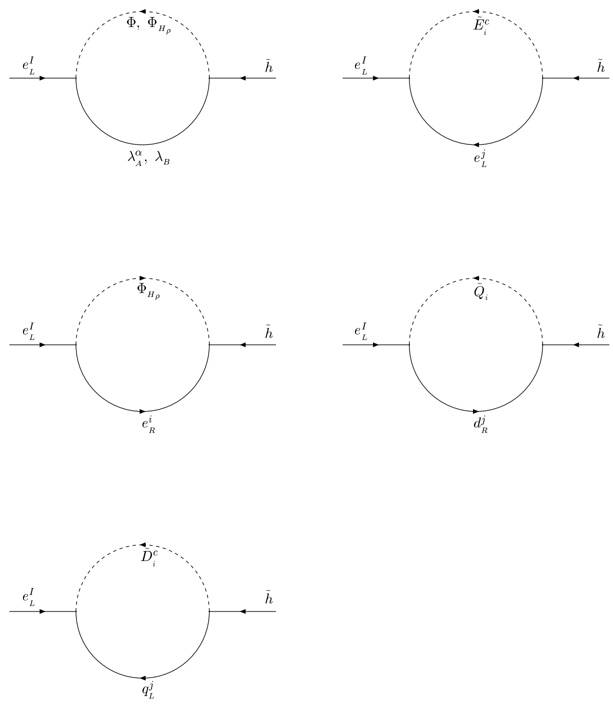

FIG. 7: The lepton higgsino self-energy diagrams that lead to the dimension-five operators after the matching.

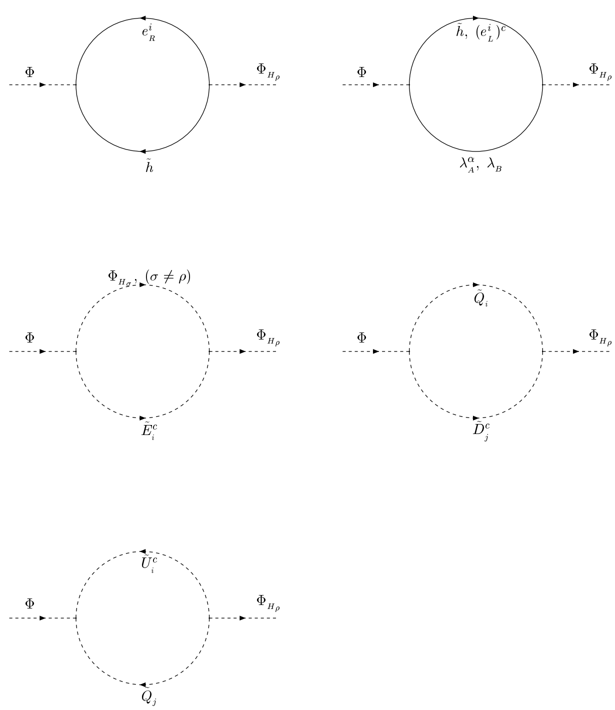

FIG. 8: The Higgs self-energy diagrams that lead to the dimension-five operators after the matching.

For the bilinear couplings we use the values

$$\begin{aligned}
&\epsilon_0 = 5 \text{ TeV}, \ \epsilon_1 = 0.01 \text{ GeV}, \\
&\epsilon_2 = 0.3 \text{ GeV}, \ \epsilon_3 = 4 \text{ GeV}, \\
&B_0 = 100 \text{ TeV}^2, \ B_1 = B_2 = B_3 = 100 \text{ GeV}^2 \ .
\end{aligned} \tag{37}$$

In order to obtain small neutrino masses, $m_\nu < 0.1$ eV, we assume the gaugino masses very large, $|m_1| = |m_2| = 100$ TeV. We consider the numerical values of the parameters given above to be conceivable and representative for the model.

In order to demonstrate the effects of CP-phases on the masses of neutrinos we plot in Fig.9, in the case $\theta_2 = arg(m_2) = 0$, the neutrino mass-squared differences Δm^2_{32}, Δm^2_{21}, and the neutrino masses m_{ν_2}, m_{ν_3} versus the CP-phase $\theta_1 = arg(m_1)$. We have considered the situation for both large ($\tan\beta = 50$, solid line) and small ($\tan\beta = 5$, dashed line) values of the ratio $\tan\beta = v_u/v_d$. In Fig.10 we present a similar plot the same for $\theta_1 = arg(m_1) = 0$ and a varying $\theta_2 = arg(m_2)$. It is seen from these plots that the solar and atmospheric neutrino data, which allow the 3σ-range shown in gray in the figures, sets non-trivial connstraints on the complex phases, in particular in the case of a small $\tan\beta$.

Let us finally note that we have found, by scanning the parameter space, that when all the trilinear $\not{L}$ couplings together with corresponding soft terms are included, it is possible to realize in this model a situation, where a large neutrino mass hierarchy is associated with a large mixing. In general, this kind of situation is not so easy to achieve in a definite theoretical framework.

V. SUMMARY

There are many ways to generate neutrino masses in the Minimal Supersymmetric Standard Model without R-parity. In this paper we have derived, using the effective field theory approach, the high-dimension operators relevant for neutrino masses, which the R-parity breaking couplings induce. Among these high-dimension operators, the dimension-odd operators originating from the lepton number violation couplings will lead to nonzero Majorana masses after the EW symmetry breaking. By properly defining the heavy fields, we have derived the Wilson coefficients of dimension-five operators at one loop level and the

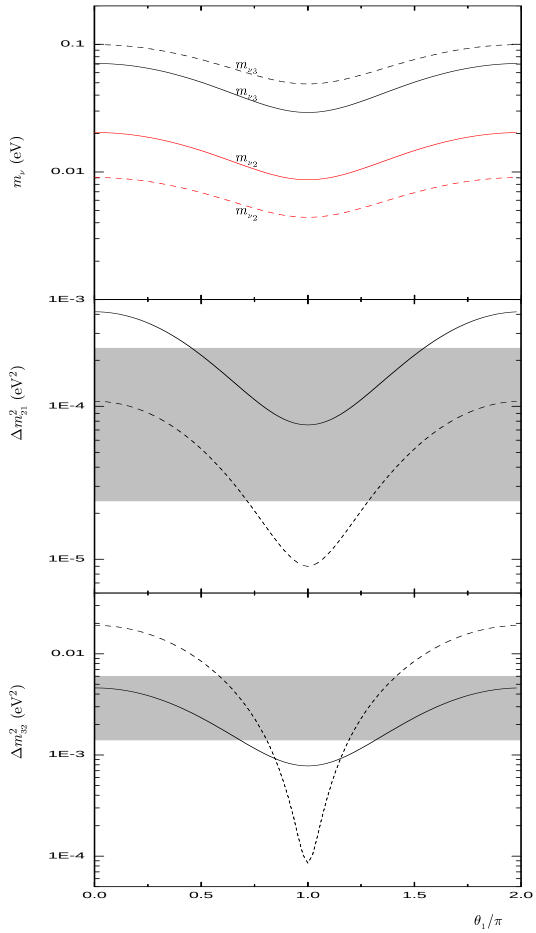

FIG. 9: The neutrino mass-squared differences Δm^2_{32}, Δm^2_{21}, and the neutrino masses m_{ν_2}, m_{ν_3} versus the CP phase $\theta_1 = arg(m_1)$ for $\theta_2 = arg(m_2) = 0$. The solid lines correspond $\tan\beta = 50$ and the dash lines to $\tan\beta = 5$. The gray bands represent the region allowed by the global fit of the solar and atmospheric neutrino data at 3σ level. For the values of the other parameters, see the text.

those of dimension-seven operators at tree level. We have demonstrated the effect of CP phases on neutrino masses by looking at a specific case, where the only complex R-parity violating couplings are the soft bilinear gaugino mass terms.

Acknowledgments

The work has been supported by the Academy of Finland under the contracts no. 104915 and 107293.

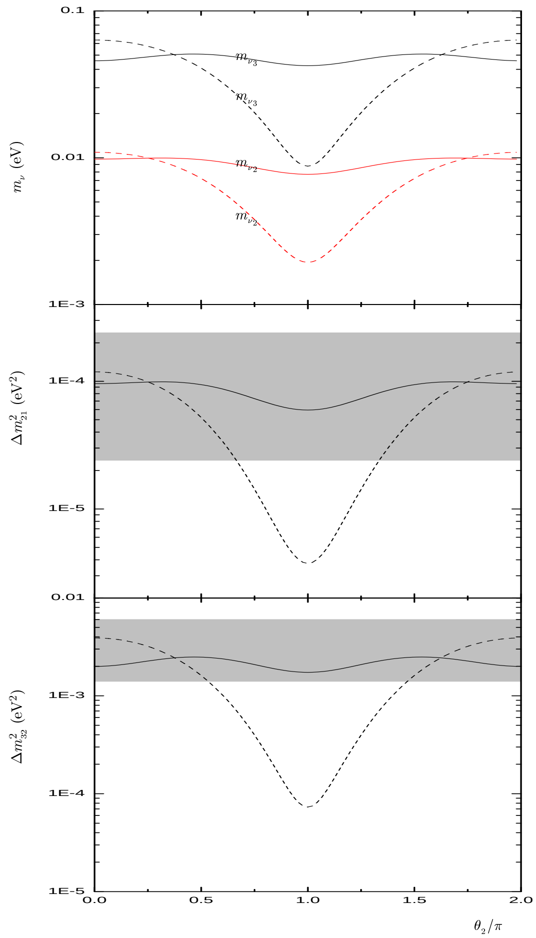

FIG. 10: The neutrino mass-squared differences Δm^2_{32}, Δm^2_{21}, and the neutrino masses m_{ν_2}, m_{ν_3} versus the CP phase $\theta_2 = arg(m_2)$ for $\theta_1 = arg(m_1) = 0$. The solid lines correspond to $\tan\beta = 50$, the dash lines $\tan\beta = 5$.

APPENDIX A: THE ONE-LOOP CORRECTIONS ON THE WILSON COEFFICIENTS OF THE DIMENSION-FIVE OPERATORS

In this appendix, we present the one-loop corrections on the Wilson coefficients of the dimension-five operators that are present in Eq. (19). The triangle diagram corrections are

$$
\begin{aligned}
(\delta C_1^{(1)})_{J,I} = & \frac{g_1^2 \mu_H}{2\mu^2_{\rm NP}|m_1|} e^{-i(\theta_\mu+\theta_1)} c^2_\beta \Big[\Lambda_{0,J}(\mathbf{Y}_{R_M})_{0,I} + \Lambda_{0,I}(\mathbf{Y}_{R_M})_{0,J}\Big](\mathcal{Z}_R)_{M,i}(\mathcal{Z}_R^\dagger)_{i,N}(\mathbf{A}_{S_N})_{0,\alpha} \\
& \times(\mathcal{Z}_H)_{\alpha,1} C(x_{\tilde h}, x_{H_\alpha}, x_{R_i}) \\
& + \frac{g_1^4}{2|m_1|} e^{-i\theta_1} c^2_\beta \Lambda_{0,I}\Lambda_{0,J}\Big[s^2_\beta B(x_{\tilde h}, x_{\tilde B}) - s_\beta c_\beta \Lambda_{0,0} e^{-i(\theta_\mu+\theta_1)}\sqrt{x_{\tilde h} x_{\tilde B}} P(x_{\tilde h}, x_{\tilde B})\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{3g_1^2g_2^2}{2|m_1|}e^{-i\theta_1}c_\beta^2\Lambda_{0,I}\Lambda_{0,J}\Big[s_\beta^2 B(x_{\tilde h},x_{\tilde A})-s_\beta c_\beta\Lambda_{0,0}e^{-i(\theta_\mu+\theta_2)}\sqrt{x_{\tilde h}x_{\tilde A}}P(x_{\tilde h},x_{\tilde A})\Big]\\
&+\frac{g_1^4}{4|m_1|}e^{-i\theta_1}c_\beta^2\Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big]\\
&\times(\mathcal{Z}_H)_{\alpha,1}\Big[s_\beta C_0(x_{H_\alpha},x_{\tilde h},x_{\tilde B})\\
&-c_\beta\Lambda_{0,0}\sqrt{x_{\tilde h}x_{\tilde B}}e^{-i(\theta_\mu+\theta_1)}C(x_{H_\alpha},x_{\tilde h},x_{\tilde B})\Big]+\frac{3g_1^2g_2^2}{4|m_1|}e^{-i\theta_1}c_\beta^2\Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha}\\
&+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big](\mathcal{Z}_H)_{\alpha,1}\Big[s_\beta C_0(x_{H_\alpha},x_{\tilde h},x_{\tilde A})\\
&-c_\beta\Lambda_{0,0}\sqrt{x_{\tilde h}x_{\tilde A}}e^{-i(\theta_\mu+\theta_2)}C(x_{H_\alpha},x_{\tilde h},x_{\tilde A})\Big]\\
&+\frac{g_1^2}{|m_1|}e^{-i\theta_1}c_\beta^2\Big[\Lambda_{0,I}(\mathbf{Y}_{L_M}^\dagger)_0(\mathbf{Y}_{R_N})_{0,J}+\Lambda_{0,J}(\mathbf{Y}_{L_M}^\dagger)_0(\mathbf{Y}_{R_N})_{0,I}\Big](\mathcal{Z}_R^\dagger)_{M,i}(\mathcal{Z}_R)_{i,N}\\
&\times B(x_{\tilde h},x_{R_i})\\
&+\frac{g_1^2}{|m_1|}e^{-i\theta_1}c_\beta^2\Big[\Lambda_{0,I}\sum_{j=1}^3(\mathbf{Y}_{L_M}^\dagger)_j(\mathbf{Y}_{R_N})_{j,J}+\Lambda_{0,J}\sum_{j=1}^3(\mathbf{Y}_{L_M}^\dagger)_j(\mathbf{Y}_{R_N})_{j,I}\Big](\mathcal{Z}_R^\dagger)_{M,i}\\
&\times(\mathcal{Z}_R)_{i,N}A(x_{R_i})\\
&+\frac{g_1^2}{3|m_1|}e^{-i\theta_1}c_\beta^2\Big[\Lambda_{0,I}(\mathbf{Y}_{D_M}\mathbf{Y}_{S_N})_{0,J}+\Lambda_{0,J}(\mathbf{Y}_{D_M}\mathbf{Y}_{S_N})_{0,I}\Big](\mathcal{Z}_D^\dagger)_{M,i}\\
&\times(\mathcal{Z}_D)_{i,N}A(x_{D_i})\\
&+\frac{g_1^2}{6|m_1|}e^{-i\theta_1}c_\beta^2\Big[\Lambda_{0,I}(\mathbf{Y}_{D_K}^\dagger)_{0,M}(\mathbf{Y}_{S_K})_{N,J}+\Lambda_{0,J}(\mathbf{Y}_{D_K}^\dagger)_{0,M}(\mathbf{Y}_{S_K})_{N,I}\Big](\mathcal{Z}_Q^\dagger)_{M,i}\\
&\times(\mathcal{Z}_Q)_{i,N}A(x_{Q_i})\ ,
\end{aligned}
$$

$$
\begin{aligned}
(\delta C_2^{(1)})_{J,I}=&\frac{g_2^2\mu_H}{2\mu_{\rm NP}^2|m_2|}e^{-i(\theta_\mu+\theta_2)}c_\beta^2\Big[\Lambda_{0,J}(\mathbf{Y}_{R_M})_{0,I}+\Lambda_{0,I}(\mathbf{Y}_{R_M})_{0,J}\Big](\mathcal{Z}_R)_{M,i}(\mathcal{Z}_R^\dagger)_{i,N}(\mathbf{A}_{S_N})_{0,\alpha}\\
&\times(\mathcal{Z}_H)_{\alpha,1}C(x_{\tilde h},x_{H_\alpha},x_{R_i})\\
&+\frac{g_2^4}{2|m_2|}e^{-i\theta_2}c_\beta^2\Lambda_{0,I}\Lambda_{0,J}\Big[s_\beta^2B(x_{\tilde h},x_{\tilde A})-s_\beta c_\beta\Lambda_{0,0}e^{-i(\theta_\mu+\theta_2)}\sqrt{x_{\tilde h}x_{\tilde A}}P(x_{\tilde h},x_{\tilde A})\Big]\\
&+\frac{g_1^2g_2^2}{2|m_2|}e^{-i\theta_2}c_\beta^2\Lambda_{0,I}\Lambda_{0,J}\Big[s_\beta^2B(x_{\tilde h},x_{\tilde B})-s_\beta c_\beta\Lambda_{0,0}e^{-i(\theta_\mu+\theta_1)}\sqrt{x_{\tilde h}x_{\tilde B}}P(x_{\tilde h},x_{\tilde B})\Big]\\
&+\frac{g_2^4}{4|m_2|}e^{-i\theta_2}c_\beta^2\Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big]\\
&\times(\mathcal{Z}_H)_{\alpha,1}\Big[s_\beta C_0(x_{H_\alpha},x_{\tilde h},x_{\tilde A})-c_\beta\Lambda_{0,0}\sqrt{x_{\tilde h}x_{\tilde A}}e^{-i(\theta_\mu+\theta_2)}C(x_{H_\alpha},x_{\tilde h},x_{\tilde A})\Big]\\
&+\frac{g_1^2g_2^2}{4|m_2|}e^{-i\theta_2}c_\beta^2\Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big]\\
&\times(\mathcal{Z}_H)_{\alpha,1}\Big[s_\beta C_0(x_{H_\alpha},x_{\tilde h},x_{\tilde B})-c_\beta\Lambda_{0,0}\sqrt{x_{\tilde h}x_{\tilde B}}e^{-i(\theta_\mu+\theta_1)}C(x_{H_\alpha},x_{\tilde h},x_{\tilde B})\Big]\\
&+\frac{g_2^2}{2|m_2|}e^{-i\theta_2}c_\beta^2\Big[\Lambda_{0,I}(\mathbf{Y}_{D_K}^\dagger)_{0,M}(\mathbf{Y}_{S_K})_{N,J}+\Lambda_{0,J}(\mathbf{Y}_{D_K}^\dagger)_{0,M}(\mathbf{Y}_{S_K})_{N,I}\Big](\mathcal{Z}_Q^\dagger)_{M,i}
\end{aligned}
$$

$$\times(\mathcal{Z}_{Q})_{i,N}A(x_{Q_i}) \tag{A1}$$

with $x_i = m_i^2/\mu_{NP}^2$.

The corrections from the self-energy diagrams

$$\begin{aligned}
(\delta C_1^{(2)})_{J,I} = & -\frac{g_1^4\mu_H}{4|m_1|^2}e^{-i(\theta_\mu+2\theta_1)}s_\beta c_\beta^3\Lambda_{0,0}\Lambda_{0,I}\Lambda_{0,J}A(x_{\tilde{h}}) \\
& -\frac{g_1^4\mu_H}{4|m_1|^2}e^{-i(\theta_\mu+2\theta_1)}c_\beta^3\Big(s_\beta\Lambda_{0,0}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Lambda_{i,0}(\mathcal{Z}_H^\dagger)_{i,\alpha}\Big)(\mathcal{Z}_H)_{\alpha,1}\Lambda_{0,I}\Lambda_{0,J}B(x_{\tilde{h}},x_{H_\alpha}) \\
& -\frac{g_1^4}{4\mu_H}e^{-i(\theta_\mu+2\theta_1)}c_\beta^3\Lambda_{0,0}\Big\{2s_\beta\Lambda_{0,I}\Lambda_{0,J}A(x_{\tilde{B}})+\Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J} \\
& +\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i,\alpha}\Big](\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{B}},x_{\tilde{h}})\Big\} \\
& -\frac{3g_1^2g_2^2|m_2|}{4\mu_H|m_1|}e^{-i(\theta_\mu+\theta_1+\theta_2)}c_\beta^3\Lambda_{0,0}\Big\{2s_\beta\Lambda_{0,I}\Lambda_{0,J}A(x_{\tilde{A}})+\Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha} \\
& +\sum_{i=1}^3\Big(\Lambda_{i,I}\Lambda_{0,J}+\Lambda_{0,I}\Lambda_{i,J}\Big)(\mathcal{Z}_H^\dagger)_{i,\alpha}\Big](\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{A}},x_{\tilde{h}})\Big\} \\
& +\frac{3g_1^2g_2^2}{2|m_1|m_{H_\alpha}^2}e^{-i\theta_1}c_\beta\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
& \times\Big\{\Big[s_\beta c_\beta\Big(1-|\Lambda_{0,0}|^2\Big)(\mathcal{Z}_H)_{\alpha,1}-c_\beta\Lambda_{0,0}\sum_{i=1}^3\Lambda_{\alpha,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B_0(x_{\tilde{h}},x_{\tilde{A}}) \\
& -c_\beta^2\mu_H|m_2|e^{-i(\theta_\mu+\theta_2)}\Lambda_{0,0}(\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{h}},x_{\tilde{A}})-s_\beta\mu_H|m_2|e^{i(\theta_\mu+\theta_2)}\Big[s_\beta\Lambda_{0,0}^*(\mathcal{Z}_H)_{\alpha,1} \\
& +\sum_{i=1}^3\Lambda_{i,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B(x_{\tilde{h}},x_{\tilde{A}})\Big\} \\
& +\frac{3g_1^2g_2^2}{2|m_1|m_{H_\alpha}^2}e^{-i\theta_1}c_\beta^2\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
& \times\sum_{j=1}^3\Lambda_{0,j}\Big[s_\beta\Lambda_{0,j}^*(\mathcal{Z}_H)_{\alpha,1}+\sum_{i=1}^3\Lambda_{i,j}^*(\mathcal{Z}_H)_{\alpha,i}\Big]A_0(x_{\tilde{A}}) \\
& +\frac{g_1^4}{2|m_1|m_{H_\alpha}^2}e^{-i\theta_1}c_\beta\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
& \times\Big\{\Big[s_\beta c_\beta\Big(1-|\Lambda_{0,0}|^2\Big)(\mathcal{Z}_H)_{\alpha,1}-c_\beta\Lambda_{0,0}\sum_{i=1}^3\Lambda_{\alpha,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B_0(x_{\tilde{h}},x_{\tilde{B}}) \\
& -c_\beta^2\mu_H|m_1|e^{-i(\theta_\mu+\theta_1)}\Lambda_{0,0}(\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{h}},x_{\tilde{B}})-s_\beta\mu_H|m_1|e^{i(\theta_\mu+\theta_1)}\Big[s_\beta\Lambda_{0,0}^*(\mathcal{Z}_H)_{\alpha,1} \\
& +\sum_{i=1}^3\Lambda_{i,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B(x_{\tilde{h}},x_{\tilde{B}})\Big\} \\
& +\frac{g_1^4}{2|m_1|m_{H_\alpha}^2}e^{-i\theta_1}c_\beta^2\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha}+\sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J}+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
& \times\sum_{j=1}^3\Lambda_{0,j}\Big[s_\beta\Lambda_{0,j}^*(\mathcal{Z}_H)_{\alpha,1}+\sum_{i=1}^3\Lambda_{i,j}^*(\mathcal{Z}_H)_{\alpha,i}\Big]A_0(x_{\tilde{B}})\ ,
\end{aligned}$$

$$
\begin{aligned}
(\delta C_2^{(2)})_{J,I} =& -\frac{g_2^4\mu_H}{4|m_2|^2}e^{-i(\theta_\mu+2\theta_2)}s_\beta c_\beta^3\Lambda_{0,0}\Lambda_{0,I}\Lambda_{0,J}A(x_{\tilde{h}}) - \frac{g_2^4\mu_H}{4|m_2|^2}e^{-i(\theta_\mu+2\theta_2)}c_\beta^3 \\
&\times\Big(s_\beta\Lambda_{0,0}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Lambda_{i,0}(\mathcal{Z}_H^\dagger)_{i,\alpha}\Big)(\mathcal{Z}_H)_{\alpha,1}\Lambda_{0,I}\Lambda_{0,J}B(x_{\tilde{h}},x_{H_\alpha}) \\
&-\frac{3g_2^4}{4\mu_H}e^{-i(\theta_\mu+2\theta_2)}c_\beta^3\Lambda_{0,0}\Big\{2s_\beta\Lambda_{0,I}\Lambda_{0,J}A(x_{\tilde{A}}) + \Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J} \\
&+\Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i,\alpha}\Big](\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{A}},x_{\tilde{h}})\Big\} - \frac{g_1^2g_2^2|m_1|}{4\mu_H|m_2|}e^{-i(\theta_\mu+\theta_1+\theta_2)}c_\beta^3\Lambda_{0,0} \\
&\times\Big\{2s_\beta\Lambda_{0,I}\Lambda_{0,J}A(x_{\tilde{B}}) + \Big[2s_\beta\Lambda_{0,I}\Lambda_{0,J}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Big(\Lambda_{i,I}\Lambda_{0,J} + \Lambda_{0,I}\Lambda_{i,J}\Big)(\mathcal{Z}_H^\dagger)_{i,\alpha}\Big] \\
&\times(\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{B}},x_{\tilde{h}})\Big\} \\
&+\frac{3g_2^4}{2|m_2|m_{H_\alpha}^2}e^{-i\theta_2}c_\beta\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J} + \Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
&\times\Big\{\Big[s_\beta c_\beta\Big(1-|\Lambda_{0,0}|^2\Big)(\mathcal{Z}_H)_{\alpha,1} - c_\beta\Lambda_{0,0}\sum_{i=1}^3\Lambda_{\alpha,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B_0(x_{\tilde{h}},x_{\tilde{A}}) \\
&-c_\beta^2\mu_H|m_2|e^{-i(\theta_\mu+\theta_2)}\Lambda_{0,0}(\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{h}},x_{\tilde{A}}) - s_\beta\mu_H|m_2|e^{i(\theta_\mu+\theta_2)}\Big[s_\beta\Lambda_{0,0}^*(\mathcal{Z}_H)_{\alpha,1} \\
&+\sum_{i=1}^3\Lambda_{i,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B(x_{\tilde{h}},x_{\tilde{A}})\Big\} \\
&+\frac{3g_2^4}{2|m_2|m_{H_\alpha}^2}e^{-i\theta_2}c_\beta^2\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J} + \Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
&\times\sum_{j=1}^3\Lambda_{0,j}\Big[s_\beta\Lambda_{0,j}^*(\mathcal{Z}_H)_{\alpha,1} + \sum_{i=1}^3\Lambda_{i,j}^*(\mathcal{Z}_H)_{\alpha,i}\Big]A_0(x_{\tilde{A}}) \\
&+\frac{g_1^2g_2^2}{2|m_2|m_{H_\alpha}^2}e^{-i\theta_2}c_\beta\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J} + \Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
&\times\Big\{\Big[s_\beta c_\beta\Big(1-|\Lambda_{0,0}|^2\Big)(\mathcal{Z}_H)_{\alpha,1} - c_\beta\Lambda_{0,0}\sum_{i=1}^3\Lambda_{\alpha,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B_0(x_{\tilde{h}},x_{\tilde{B}}) \\
&-c_\beta^2\mu_H|m_1|e^{-i(\theta_\mu+\theta_1)}\Lambda_{0,0}(\mathcal{Z}_H)_{\alpha,1}B(x_{\tilde{h}},x_{\tilde{B}}) - s_\beta\mu_H|m_1|e^{i(\theta_\mu+\theta_1)}\Big[s_\beta\Lambda_{0,0}^*(\mathcal{Z}_H)_{\alpha,1} \\
&+\sum_{i=1}^3\Lambda_{i,0}^*(\mathcal{Z}_H)_{\alpha,i+1}\Big]B(x_{\tilde{h}},x_{\tilde{B}})\Big\} \\
&+\frac{g_1^2g_2^2}{2|m_2|m_{H_\alpha}^2}e^{-i\theta_2}c_\beta^2\Big[2s_\beta\Lambda_{0,J}\Lambda_{0,I}(\mathcal{Z}_H^\dagger)_{1,\alpha} + \sum_{i=1}^3\Big(\Lambda_{0,I}\Lambda_{i,J} + \Lambda_{i,I}\Lambda_{0,J}\Big)(\mathcal{Z}_H^\dagger)_{i+1,\alpha}\Big] \\
&\times\sum_{j=1}^3\Lambda_{0,j}\Big[s_\beta\Lambda_{0,j}^*(\mathcal{Z}_H)_{\alpha,1} + \sum_{i=1}^3\Lambda_{i,j}^*(\mathcal{Z}_H)_{\alpha,i}\Big]A_0(x_{\tilde{B}})\ ,
\end{aligned}
\tag{A2}
$$

where $x_i = m_i^2/\mu_{\rm NP}^2$, and the nonzero Yukawa couplings are

$$
\begin{aligned}
(\mathbf{Y}_{R_K})_{0,1} &= h^e_{1,K}\frac{\mu_{H_1}}{\mu_H} + \frac{\Big(h^e_{2,K}|\epsilon_1|e^{i\varphi_1} - 2\lambda_{12K}|\epsilon_0|\Big)|\epsilon_2|e^{-i\varphi_2}}{\mu_H\mu_{H_1}} \\
&\quad + \frac{\Big(h^e_{3,K}|\epsilon_1|e^{i\varphi_1} - 2\lambda_{13K}|\epsilon_0|\Big)|\epsilon_3|e^{-i\varphi_3}}{\mu_H\mu_{H_1}} \ , \\
(\mathbf{Y}_{R_K})_{0,2} &= \frac{\mu_{H_2}\Big(h^e_{2,K}|\epsilon_0| - 2\lambda_{21K}|\epsilon_1|e^{-i\varphi_1}\Big)}{\mu_{H_1}\mu_H} + \frac{|\epsilon_2\epsilon_3|e^{i(\varphi_2-\varphi_3)}\Big(h^e_{3,K}|\epsilon_0| + 2\lambda_{13K}|\epsilon_1|e^{-i\varphi_1}\Big)}{\mu_{H_1}\mu_{H_2}\mu_H} \\
&\quad -2\lambda_{23K}\frac{\mu_{H_1}|\epsilon_3|}{\mu_{H_2}\mu_H}e^{-i\varphi_3} \ , \\
(\mathbf{Y}_{R_K})_{0,3} &= \frac{1}{\mu_{H_2}}\Big\{h^e_{3,K}|\epsilon_0| - 2\Big(\lambda_{31K}|\epsilon_1|e^{-i\varphi_1} + \lambda_{32K}|\epsilon_2|e^{-i\varphi_2}\Big)\Big\} \ , \\
(\mathbf{Y}_{R_K})_{1,2} &= \frac{h^e_{1,K}|\epsilon_2|e^{i\varphi_2} - h^e_{2,K}|\epsilon_1|e^{i\varphi_1}}{\mu_{H_2}} + 2\lambda_{12K}\frac{|\epsilon_0|}{\mu_{H_2}} \ , \\
(\mathbf{Y}_{R_K})_{1,3} &= h^e_{1,K}\frac{\mu_{H_1}|\epsilon_3|}{\mu_{H_2}\mu_H}e^{i\varphi_3} + \frac{|\epsilon_2\epsilon_3|e^{i(\varphi_3-\varphi_2)}\Big(h^e_{2,K}|\epsilon_1|e^{-i\varphi_1} - 2\lambda_{12K}|\epsilon_0|\Big)}{\mu_{H_1}\mu_{H_2}\mu_H} \\
&\quad - \frac{\mu_{H_2}\Big(h^e_{3,K}|\epsilon_1|e^{-i\varphi_1} - 2\lambda_{13K}|\epsilon_0|\Big)}{\mu_{H_1}\mu_H} \ , \\
(\mathbf{Y}_{R_K})_{2,3} &= 2\lambda_{23K}\frac{\mu_{H_1}}{\mu_H} + \frac{|\epsilon_3|e^{i\varphi_3}\Big(h^e_{2,K}|\epsilon_0| + 2\lambda_{12K}|\epsilon_1|e^{-i\varphi_1}\Big)}{\mu_{H_1}\mu_H} \\
&\quad - \frac{|\epsilon_2|e^{i\varphi_2}\Big(h^e_{3,K}|\epsilon_0| + 2\lambda_{13K}|\epsilon_1|e^{-i\varphi_1}\Big)}{\mu_{H_1}\mu_H} \ , \\
(\mathbf{Y}_{L_K})_0 &= \sum_{I=1}^{3}\frac{\Big(\upsilon_0|\epsilon_I|e^{-i\varphi_I} - \upsilon_I|\epsilon_0|\Big)h^e_{I,K}}{\upsilon_d\mu_H} + \sum_{I,J}\frac{\Big(\upsilon_J|\epsilon_I|e^{-i\varphi_I} - \upsilon_I|\epsilon_J|e^{-i\varphi_J}\Big)}{\upsilon_d\mu_H}\lambda_{IJK} \ , \\
(\mathbf{Y}_{L_K})_1 &= \frac{|\epsilon_0|\mathcal{H}^e_{1,K} + \sum\limits_{I=1}^{3}\upsilon_I h^e_{I,K}|\epsilon_1|e^{i\varphi_1}}{\upsilon_d\mu_{H_1}} \ , \\
(\mathbf{Y}_{L_K})_2 &= \frac{1}{\upsilon_d\mu_{H_1}\mu_{H_2}}\Big\{\mu^2_{H_1}\mathcal{H}^e_{2,K} - \mathcal{H}^e_{1,K}|\epsilon_1\epsilon_2|e^{i(\varphi_2-\varphi_1)} + \sum_{I=1}^{3}\upsilon_I h^e_{I,K}|\epsilon_0\epsilon_2|e^{i\varphi_2}\Big\} \ , \\
(\mathbf{Y}_{L_K})_3 &= \frac{1}{\upsilon_d\mu_{H_2}\mu_H}\Big\{\mu^2_{H_2}\mathcal{H}^e_{3,K} - \mathcal{H}^e_{1,K}|\epsilon_1\epsilon_3|e^{i(\varphi_3-\varphi_1)} - \mathcal{H}^e_{2,K}|\epsilon_2\epsilon_3|e^{i(\varphi_3-\varphi_1)} \\
&\quad + \sum_{I=1}^{3}\upsilon_I h^e_{I,K}|\epsilon_0\epsilon_3|e^{i\varphi_3}\Big\} \ , \\
(\mathbf{Y}_{D_K})_{0,I} &= \frac{\upsilon_0 h^d_{I,K} + \sum\limits_{J=1}^{3}\upsilon_J\lambda'_{JIK}}{\upsilon_d} \ ,
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{Y}_{S_K})_{I,1} &= \frac{h^d_{I,K}|\epsilon_1|e^{i\varphi_1} - \lambda'_{1IK}|\epsilon_0|}{\mu_{H_1}} \;, \\
(\mathbf{Y}_{S_K})_{I,2} &= \frac{h^d_{I,K}|\epsilon_0\epsilon_2|e^{i\varphi_2} + \lambda'_{1IK}|\epsilon_1\epsilon_2|e^{i(\varphi_2-\varphi_1)}}{\mu_{H_1}\mu_{H_2}} - \frac{\mu_{H_1}\lambda'_{2IK}}{\mu_{H_2}} \;, \\
(\mathbf{Y}_{S_K})_{I,3} &= \frac{1}{\mu_{H_2}\mu_H}\Big\{h^d_{I,K}|\epsilon_0\epsilon_3 e^{i\varphi_3} + \lambda'_{1IK}|\epsilon_1\epsilon_3|e^{i(\varphi_3-\varphi_1)} + \lambda'_{2IK}|\epsilon_2\epsilon_3|e^{i(\varphi_3-\varphi_2)} \\
&\quad -\mu^2_{H_2}\lambda'_{3IK}\Big\} \qquad \text{(A3)}
\end{aligned}
$$

with

$$
\mathcal{H}^e_{I,J} = \upsilon_0 h^e_{I,J} + 2\sum_{\rho\neq I}\upsilon_\rho\lambda_{\rho IJ} \;. \qquad \text{(A4)}
$$

The couplings between the Higgs and right-handed slepton are given by

$$
(\mathbf{A}_{S_I})_{0,\alpha} = s^2_\beta\sum_{\rho=1}^{3}\frac{\upsilon_\rho}{\upsilon_d}\eta^I_\rho(\mathcal{Z}_H)_{\alpha,0} + \sum_{K=1}^{3}\Big[s_\beta\frac{\upsilon^2_{d_{K-1}}\eta^I_K - \upsilon_K\sum\limits_{\beta=0}^{K-1}\upsilon_\beta\eta^I_\alpha}{\upsilon_{d_{K-1}}\upsilon_{d_K}} + c_\beta\xi^I_K\Big](\mathcal{Z}_H)_{\alpha,K} \qquad \text{(A5)}
$$

with

$$
\begin{aligned}
\eta^I_0 &= \sum_{K=1}^{3}\epsilon^*_K h^e_{K,I}, \\
\eta^I_K &= -\epsilon^*_0 h^e_{K,I} - 2\sum_{J=1}^{3}\epsilon^*_J\lambda_{JKI}, \\
\xi^I_1 &= \frac{\upsilon_{d_1}}{\upsilon_d}A^e_{1I} + \frac{\upsilon_2(\upsilon_1 A^e_{2I} - 2\upsilon_0 A_{12I})}{\upsilon_{d_1}\upsilon_d} + \frac{\upsilon_3(\upsilon_1 A^e_{3I} - 2\upsilon_0 A_{13I})}{\upsilon_{d_1}\upsilon_d}, \\
\xi^I_2 &= \frac{\upsilon_{d_2}(\upsilon_0 A^e_{2I} + 2\upsilon_1 A_{12I})}{\upsilon_{d_1}\upsilon_d} - 2\frac{\upsilon_3\upsilon_{d_1}}{\upsilon_{d_2}\upsilon_d}A_{23I} + \frac{\upsilon_0\upsilon_2\upsilon_3}{\upsilon_{d_1}\upsilon_{d_2}\upsilon_d}A^e_{3I} + 2\frac{\upsilon_1\upsilon_2\upsilon_3}{\upsilon_{d_1}\upsilon_{d_2}\upsilon_d}A_{13I}, \\
\xi^I_3 &= \frac{\upsilon_0 A^e_{3I} + 2\upsilon_1 A_{13I} + 2\upsilon_2 A_{23I}}{\upsilon_{d_2}} \;. \qquad \text{(A6)}
\end{aligned}
$$

The loop-integral functions are

$$
\begin{aligned}
A(x) &= \frac{1-\ln x}{(4\pi)^2} \;, \\
A_0(x) &= x\frac{1-\ln x}{(4\pi)^2} \;, \\
B(x,y) &= \frac{1}{(4\pi)^2}\Big[1 + \frac{x\ln x}{y-x} + \frac{y\ln y}{x-y}\Big] \;, \\
B_0(x,y) &= \frac{1}{(4\pi)^2}\Big[x + y + \frac{x^2\ln x}{y-x} + \frac{y^2\ln y}{x-y}\Big] \;, \\
P(x,y) &= \frac{1}{(4\pi)^2}\Big[\frac{\ln x}{x-y} + \frac{\ln y}{y-x}\Big] \;,
\end{aligned}
$$

$$
\begin{aligned}
C(x,y,z) &= \frac{1}{(4\pi)^2}\Big[\frac{x\ln x}{(y-x)(z-x)} \\
&+\frac{y\ln y}{(x-y)(z-y)}+\frac{z\ln z}{(x-z)(y-z)}\Big]\;, \\
C_0(x,y,z) &= \frac{1}{(4\pi)^2}\Big[\frac{x^2\ln x}{(y-x)(z-x)} \\
&+\frac{y^2\ln y}{(x-y)(z-y)}+\frac{z^2\ln z}{(x-z)(y-z)}-1\Big]\;. \qquad \text{(A7)}
\end{aligned}
$$

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